

**An alternate
background noise
correction sensitive to
the steadiness of
background noise**

Technical
Report



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Introduction

This Technical Report presents an alternate background noise correction sensitive to the steadiness of background noise. The alternate background noise correction is used in the same manner as the standard background noise corrections in ISO 3741, ISO 3744, ISO 3745, and ISO 11201 and can be applied to A-weighted or un-weighted band or overall sound power or sound pressure levels.

The purpose of a background noise correction is to remove a background noise contribution from a measured source level, which contains source and background noise contributions. The result is a true source estimate called the background noise corrected source level. The background noise contribution to the measured source level must be estimated, since measured source and background noise levels are measured over distinct and separate time spans. Since background noise fluctuates over time, a background noise corrected source level has uncertainty and may overstate or understate the true source level.

Present standards manage the uncertainty by imposing caps on background noise corrections, the idea being to minimize risk of understating true source level. The applicable standards heuristically cap the background noise correction at 0,46 dB or 1,26 dB, depending on background noise proximity to measured source level, frequency bandwidth, and chamber accuracy grade. An undesirable consequence is that background noise corrected source levels are not consistent, varying by the standard being followed. Even worse, a chamber precision penalty may occur such that background noise corrected source levels for engineering accuracy Grade 2 chambers are lower than for precision accuracy Grade 1 chambers, thereby discouraging use of high precision chambers by manufacturers. Another drawback is that the present standard background noise corrections do not statistically bound the true source level in any stated manner.

The alternate background noise correction answers these shortcomings through a statistical formulation that manages uncertainty. The alternate background noise correction produces a background corrected source level that upper bounds the true source level with 95% confidence. The steadiness of the measured background noise affects the alternate background noise correction, the magnitude of the correction tending to increase with background noise steadiness and proximity to measured source level.

The statistical formulation of the alternate background noise correction has several advantages over the standard background noise corrections.

One advantage is the certainty provided by the statistical formulation, which upper bounds true source level with 95% confidence. Statements about validity accompany the standard background noise corrections but no statistical bounds are given. Present standards deem background corrected source levels obtained from capped corrections to be invalid yet reportable; the report shows these source descriptions to upper bound the true source with unknown confidence. Background corrected source levels obtained from uncapped corrections are deemed valid, even though these source descriptions have high 50% risk of understating the true source, as shown in the report. It is expected that manufacturers and customers will appreciate the source descriptions provided by the alternate background noise correction because they upper bound the true source with known confidence.

Another advantage is that the alternate background noise correction can reduce background corrected source levels by 1 to 2 dB relative to standard corrections that are capped because of background noise within 6 to 10 dB of measured source level. The reduction of background corrected source levels relative to capped standard background noise corrections depends on background noise steadiness and proximity to measured source level, as well as the standard being followed. A statistical margin term in the alternate background noise correction based on a characterization of measured background noise allows the additional reduction. It is expected that manufacturers will find the alternate background noise correction appealing because it provides background corrected source levels that are minimized while also statistically bounding the true source.

Finally, the alternate background noise correction has the appeal of providing a path towards eliminating the inconsistent background noise corrections and the chamber precision penalty in the applicable standards. By being sensitive to both the steadiness and proximity of background noise to measured source level, the alternate background noise correction upper bounds the true source level in a manner that is applicable to background noise conditions that are stationary across measured source and background noise sampling,

without recourse to heuristic caps that can vary by standard. By adopting the alternate background noise correction in place of the standard corrections, future standards will prescribe background corrected source levels for various chambers, frequency weightings and bandwidths that are consistent and comparable to one another and free of the chamber precision penalty.

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An alternate background noise correction sensitive to the steadiness of background noise

1 Scope

This Technical Report describes an alternate background noise correction that depends not only on mean background noise proximity to measured source levels, like the standard background noise corrections of ISO 3741, ISO 3744, ISO 3745, and ISO 11201, but also on the steadiness of the background noise. Like the standard background noise corrections, the alternate background noise correction tends to increase as the background noise level approaches the measured source level, but the alternate background noise correction also increases with the steadiness of the background noise, unlike the standard background noise corrections.

2 References

For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies. Full citations of the references are available in the Bibliography.

ISO 3741:2010, *Acoustics — Determination of sound power levels and sound energy levels of noise sources using sound pressure — Precision methods for reverberation test rooms*

ISO 3744:2010, *Acoustics — Determination of sound power levels and sound energy levels of noise sources using sound pressure — Engineering methods for an essentially free field over a reflecting plane*

ISO 3745:2012, *Acoustics — Determination of sound power levels and sound energy levels of noise sources using sound pressure — Precision methods for anechoic rooms and hemi-anechoic rooms*

ISO 11201:2010, *Acoustics — Noise emitted by machinery and equipment — Determination of emission sound pressure levels at a work station and at other specified positions in an essentially free field over a reflecting plane with negligible environmental corrections*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1

background noise

average of the square of the sound pressure of background noise over time (mean square sound pressure)

NOTE Background noise is expressed in square pascals (Pa²).

3.2

background noise sample difference

difference of two samples of background noise.

NOTE Background noise sample difference is expressed in square pascals (Pa²).

3.3

background noise level

level of the background noise referenced to 20 micro-Pascals.

NOTE Background noise is expressed in decibels.

3.4 background noise corrected source level

a true source level estimate obtained by removing an estimate of the background noise contribution to the measured source level

NOTE Background noise corrected source level is expressed in decibels.

3.5 background noise correction, K

the amount by which measured source level is reduced to obtain background noise corrected source level

NOTE Background noise correction is expressed in decibels.

3.6 mean square

average of the square of sound pressure over time

NOTE Measured source level is expressed in square pascals (Pa²).

3.7 measured source level

a measured noise level containing source and background noise contributions

NOTE Measured source level is expressed in decibels.

3.8 true source level

the sound level radiated by a source, without background noise contribution and free of measurement error

NOTE True source level is expressed in decibels.

4 Abbreviations

BNCSL background noise corrected source level

VCS variance Chi-square

5 Background noise corrected source level

The background noise corrected source level L_p is an estimate of the true source level and is given by:

$$L_p = L'_p - K(\Delta L) \quad (1)$$

where L'_p is a measured source level containing both source and background noise contributions, and K is a background noise correction. The background noise correction depends on the measured source-background noise level difference $\Delta L = L'_p - L''_p$, in which $L''_p = 10 \lg(p^2 / p_0^2)$ is background noise level. The background noise level contains a ratio of the background noise p^2 to the square of the reference pressure $p_0 = 20$ micro-Pa. The background noise is the average of the square of background noise sound pressure over time.

6 Standard background noise correction

The standard background noise correction K_1 is given by

$$K_1 = \begin{cases} -10\lg(1 - 10^{-\Delta L/10}) & \Delta L > \Delta_0 \\ K_{1\max} & \Delta L \leq \Delta_0 \end{cases} \quad (2)$$

The cap $K_{1\max}$ limits the maximum value of the background noise correction to its value at Δ_0 , which is called the limitation for background noise. Noise source measurements are deemed valid by the applicable standards when $\Delta L > \Delta_0$ and the background noise correction is not capped. The values of $K_{1\max}$ and Δ_0 vary by standard depending on frequency bandwidth and grade of accuracy [1,2,3,4]. In some standards $\Delta_0 = 6$ dB and $K_{1\max} = 1,26$ dB; in other standards $\Delta_0 = 10$ dB and $K_{1\max} = 0,46$ dB. The different caps may produce background noise corrected source levels that are not consistent across the standards. Moreover, the larger cap is prescribed for engineering accuracy Grade 2 chambers and the smaller cap is prescribed for precision accuracy Grade 1 chambers, such that the Grade 1 chamber may yield a higher background noise corrected source level than the Grade 2 chamber. The situation amounts to a chamber precision penalty that may discourage use of Grade 1 chambers by manufacturers, since a lower background noise corrected source level may be obtained in a Grade 2 chamber of lower precision.

In implementing the standard background noise correction, mean values are typically used for the measured source level L'_p and the measured background noise level L''_p . No rigorous consideration is given to the fluctuation of background noise over time. A simple thought experiment suggests that background noise correction should increase with steadiness of background noise.

NOTE The symbols $K_{1\max}$ and Δ_0 have been introduced in this report for clarity; they are not found in the applicable standards.

7 Alternate background noise correction

The alternate background noise correction depends not only on the mean of the background noise but also the steadiness of the background noise. The alternate background noise correction K_{ALT} is given by

$$K_{ALT} = \min[K_U, \max(K_L, K_S)] \quad (3)$$

in which K_U and K_L are upper and lower limits imposed on statistical background noise correction K_S . For example to limit K_{ALT} values to those of the standard correction K_1 , K_U would be set to $K_{1\max}$ and K_L would be set to zero dB. Selection of these upper and lower limits must be done carefully, however, for two reasons. First, the limits change the confidence in the upper bound on the true source level provided by statistical background noise correction K_S . The upper limit K_U tends to increase upper bound confidence, and the lower limit K_L tends to decrease upper bound the confidence. The latter confidence decrease may be a concern because of the risk of understating true source level. Second, setting the upper limit to a low value, for example the standard background noise correction cap $K_{1\max}$, minimizes an advantage of the alternate background noise correction, namely to produce background corrected source levels lower than that yielded by the standard background noise correction. Specification of the upper and lower limits K_U and K_L in Equation (3) is left for future work.

The statistical background noise correction is given by

$$K_S = -10\lg\left[1 - 10^{-\Delta/10}\left(1 - \frac{u_\alpha}{2M}\right)\right] \quad (4)$$

in which u_α is a percentile of the background noise sample difference distribution and M is a background noise steadiness, as explained in Annex A. A background corrected source level produced by the statistical background noise correction K_S upper bounds the true source level with confidence value α at which the percentile is evaluated. The percentile u_α is given by

$$u_\alpha = \Phi_{VCS}^{-1}(M, \alpha) \approx \sqrt{8M} \Phi_N^{-1}(\alpha) \quad (5)$$

in which Φ_{VCS}^{-1} is the inverse cumulative VCS distribution, and the right hand side is an approximation involving the standard inverse cumulative normal distribution Φ_N^{-1} with zero mean and unit standard deviation. The background noise sample difference arises because a background noise corrected source level involves a difference between two samples of background noise. One sample is the background noise contribution to the measured source level; the other sample is the background noise removed from the measured source level by the background noise correction. Annex A shows that the background noise sample difference follows the variance Chi-square (VCS) distribution, which may be approximated by the normal distribution.

The statistical background noise correction K_S reduces to the uncapped standard background noise correction K_1 at the 50% confidence value as may be seen by comparing Equations (2) and (4). The percentile $u_\alpha = 0$ at $\alpha = 50\%$ because of the symmetry of the background noise sample difference distribution. A background noise corrected source level produced by the standard background noise correction K_1 , when uncapped, is therefore equally likely to understate or overstate the true source level. A 50% understatement risk can be undesirable especially as background noise level approaches measured source level and is the reason for the heuristic cap K_{1max} in the standard background noise correction. The cap decreases the background noise correction and increases the background corrected source level along with the confidence of upper bounding the true source level, although this effect is not expressed or quantified in the present standards.

By contrast, the statistical background noise correction manages the situation of comparable source and background noise levels through the margin term containing percentile u_α in Equation (4). The background noise correction decreases with background noise unsteadiness and proximity to measured source level (and increases with background noise steadiness and separation from measured source level).

The alternate background noise correction takes variation of the background noise into account through the background noise steadiness M :

$$M = \frac{\hat{\mu}_{p^2}^2}{\hat{\sigma}_{p^2}^2} \quad (6)$$

Here $\hat{\mu}_{p^2}$ and $\hat{\sigma}_{p^2}^2$ are measured estimates of the mean and standard deviation of the background noise obtained by sampling background noise when the source is not operating. It is important to note that the statistics are taken over the background noise—not the background noise level. Each background noise sample is an average over time of the square of the background noise sound pressure. Given a set of background noise levels L_{pi}'' ($i = 1 \dots N$) with reference value $p_0 = 20$ micro-Pa and corresponding background noise samples $p_i^2 = p_0^2 10^{L_i''/10}$, the estimates of the mean and variance are

$$\hat{\mu}_{p^2} = \frac{1}{N} \sum_{i=1}^N p_i^2 \quad (7)$$

$$\hat{\sigma}_{p^2}^2 = \frac{1}{N-1} \sum_{i=1}^N (p_i^2 - \hat{\mu}_{p^2})^2 \quad (8)$$

Plots of the VCS distribution of background noise sample difference, along with the normal approximation, are shown for various values of background noise steadiness M in Figure 1. The normal approximation nearly

matches the VCS density for $M \geq 10$. Exact VCS and approximate normal percentiles u_α are compared in Figure 2 for the confidence value $\alpha = 95\%$ selected for the alternate background noise correction. The normal approximation overstates the VCS percentile by less than 1% at $M = 1$, and the overstatement decreases rapidly with increasing M . The accuracy of the normal approximation, Equation (5), is fortunate for implementation of the alternate background noise correction because of the familiarity of the normal distribution.

NOTE In calculating the background noise steadiness M , the level reference $p_0 = 20 \mu\text{Pa}$ is unimportant because of cancellation in the numerator and denominator of Equation (6).



Annex A (normative)

Derivation of the Statistical Background Noise Correction

The derivation below follows that of an available reference^[5] except that some nomenclature and symbols have been changed for increased compatibility with existing standards. Also, the derivation has no guard band and is simpler than that of the reference^[5], while remaining mathematically equivalent.

A.1 The Problem

A source measurement m^2 has simultaneous contributions of the true source s^2 and background noise p_1^2 :

$$m^2 = s^2 + p_1^2 \quad (9)$$

Here the subscript "1" denotes the background noise sample included in the source measurement. Each of the terms in Equation (9) is a mean square, namely an average of the square of sound pressure over time. The true source may be estimated by removing a background noise estimate obtained from a separate measurement, or sample, of the background noise obtained when the source is not operating:

$$\begin{aligned} \hat{s}^2 &= m^2 - p_2^2 \\ &= s^2 + (p_1^2 - p_2^2) \end{aligned} \quad (10)$$

in which \hat{s}^2 is an estimate of the source and p_2^2 is a separately measured background noise sample. Since background noise samples p_1^2 and p_2^2 are distinct and different, their difference may be positive or negative. Source estimate \hat{s}^2 may therefore either be above or below the true source s^2 and has an accuracy that depends on the statistical behaviour of the background noise sample difference $p_1^2 - p_2^2$, which is discussed in the next section.

A.2 Background noise distribution

A statistical distribution is derived from a background noise model well established in the literature. Background noise sound pressure p is represented as an aggregation of sinusoids of various frequencies:

$$p = \sum_{m=1}^M p_m \quad (11)$$

Each sinusoid p_m is taken to be the resultant of multiple contributions of like frequency and random amplitude and phase

$$p_m = \sum_{n=1}^{N_m} A_{nm} \cos(\omega_m t + \phi_{nm}) \quad (12)$$

Here m indexes frequency components and n indexes phased sinusoidal contributions at like frequency ω_m . The amplitudes A_{nm} and phases ϕ_{nm} are random and independent of one another, the latter being uniformly distributed over $(0, 2\pi)$. The number M of frequencies and the number N_m of phase contributions at each frequency are assumed large in a statistical sense. The background noise is the mean square of background

noise sound pressure p and is obtained by combining, squaring and time-averaging the expressions in Equation (11). Since each sound pressure component p_m has unique frequency, the mean square of Equation (11) has the simple form

$$p^2 = \sum_{m=1}^M p_m^2 \quad (13)$$

Consider the average of a term p_m^2 in Equation (13) over a time scale longer than the period $2\pi/\omega_m$ but shorter than the time scale of ambient fluctuation. Squaring Equation (12) produces a double summation with terms containing squared terms $\cos^2(\omega_m t)$ and $\sin^2(\omega_m t)$ and cross terms $\sin(\omega_m t)\cos(\omega_m t)$. Averaging reduces the squared terms to one-half and zeros the cross terms, giving

$$\frac{2p_m^2}{N_m \sigma_\xi^2} = \left(\frac{\sum_n A_{nm} \cos \phi_{nm}}{\sqrt{N_m} \sigma_\xi} \right)^2 + \left(\frac{\sum_n A_{nm} \sin \phi_{nm}}{\sqrt{N_m} \sigma_\xi} \right)^2 \quad (14)$$

where σ_ξ^2 is the variance of the random variables $\xi_{1n} = A_{nm} \cos \phi_{nm}$ and $\xi_{2n} = A_{nm} \sin \phi_{nm}$. The normalization allows identification of the parenthetical quantities $\sum_n \xi_{kn} / (N_m^{1/2} \sigma_\xi)$, $k=1,2$ as normally distributed variables with zero mean and unit variance, by the Central Limit Theorem [6]. Moreover, the variable $2p_m^2 / (N_m \sigma_\xi^2)$ follows a Chi-Square distribution with 2 degrees of freedom, by definition of the Chi Square distribution [6]. The variance $\sigma_\xi^2 = \text{Var}[\xi_{kn}] = \text{Var}[A_{nm} \cos \phi_{nm}] = E(A_{nm}^2 \cos^2 \phi_{nm}) - [E(A_{nm} \cos \phi_{nm})]^2$. Here $E(\cdot)$ is the expectation operator and $\text{Var}(\cdot)$ is the variance operator. Since the amplitude A_{nm} and phase ϕ_{nm} are independent, the variance is $E(A_{nm}^2)E(\cos^2 \phi_{nm}) - [E(A_{nm})E(\cos \phi_{nm})]^2$. Defining $\overline{A_m^2} \equiv E(A_{nm}^2)$ and noting $E(\cos^2 \phi_{nm}) = \frac{1}{2}$ and $E(\cos \phi_{nm}) = 0$ leads to $\sigma_\xi^2 = \frac{1}{2} \overline{A_m^2}$. The foregoing yields

$$y_m \equiv \frac{4p_m^2}{N_m \overline{A_m^2}} \sim \chi_2^2 \quad (15)$$

In other words, the average of the square of a sinusoid made of multiple contributions of like frequency and independent random amplitude and phase follow a Chi-Square distribution χ_2^2 with two degrees of freedom, which happens to be an exponential distribution [6]. This result has been established for room and ocean acoustics [7,8].

Since background noise is typically broadband, not tonal, we now seek the distribution of the aggregation of the multiple sinusoidal components, each of unique frequency and comprised of multiple contributions with various phases, expressed in Equation (13). Also using Equation (15) reveals

$$p^2 = \sum_{m=1}^M p_m^2 = \frac{1}{4} \sum_{m=1}^M N_m \overline{A_m^2} y_m \quad (16)$$

Each component p_m^2 has unique frequency and results from multiple randomly phased sinusoidal components. If the number N_m and mean square amplitude $\overline{A_m^2}$ vary such that the product $N_m \overline{A_m^2}$ varies slowly across the analysis frequency resolution, an approximation to the product may be removed from the summation giving

$$w \equiv \frac{4p^2}{N \overline{A^2}} \approx \sum_{m=1}^M y_m \sim \chi_{2M}^2 \quad (17)$$

in which $N \overline{A^2}$ is the average of the product $N_m \overline{A_m^2}$ over frequency, and the closed property of the Chi-Square distribution has been used: $\chi_i^2 + \chi_j^2 = \chi_{(i+j)}^2$. The foregoing reveals that background noise p^2 follows

approximately a Chi-Square distribution with $\nu = 2M$ degrees of freedom, where M is the number of frequency components in the background noise.

A.3 Background noise statistics

The background noise steadiness may be obtained by Equation (17) and the properties of a Chi-Square random variable. Since a degree ν Chi-Square random variable has mean ν and variance 2ν [6], the steadiness is given by

$$M = \frac{E(\chi_{2M}^2)^2}{\text{Var}(\chi_{2M}^2)} = \frac{\mu_{p^2}^2}{\sigma_{p^2}^2} \approx \frac{\hat{\mu}_{p^2}^2}{\hat{\sigma}_{p^2}^2} \quad (18)$$

in which μ_{p^2} and $\hat{\mu}_{p^2}$ are the true and estimated mean of the background noise, and $\sigma_{p^2}^2$ and $\hat{\sigma}_{p^2}^2$ are the true and estimated variance of the background noise. The steadiness parameter is also the number of frequency components in the background noise by Equation (11). The steadiness may be determined experimentally from measured estimates of the mean and variance of background noise, as the right hand side of Equation (18) shows. Note that the statistics apply to the background noise, which is a mean square, not the background noise level.

A.4 Distribution of the background noise sample difference

Assuming background noise samples are drawn from the same background noise process, the background noise difference $p_1^2 - p_2^2$ involves two Chi-Square distributed variables with identical degrees of freedom. For statistically independent samples, the difference follows a special case of the Variance-Gamma distribution. This may be seen using the moment generating function (MGF) $M_w = E(e^{tw})$ [6]. The difference

$$u \equiv w_1 - w_2 = \frac{2M}{\mu_{p^2}} (p_1^2 - p_2^2) \quad (19)$$

has a MGF of $M_u = E[e^{t(w_2 - w_1)}] = E(e^{tw_2})E(e^{-tw_1}) = M_{w_2}(t)M_{w_1}(-t)$ since w_1 and w_2 are independent. For a Chi-Square variable of degree ν , the MGF is $M_w = (1 - 2t)^{-\nu/2}$ [6] and

$$M_u(t) = (1 - 4t^2)^{-\nu/2} = \left(\frac{1/4}{1/4 - t^2} \right)^{\nu/2} \quad (20)$$

This expression turns out to be a special case of the Variance-Gamma distribution [9], which has probability density function (PDF) $\phi_{VG}(x)$ and MGF $M_{VG}(t)$

$$\phi_{VG}(x) = \frac{(\alpha^2 - \beta^2)^\lambda}{\sqrt{\pi}\Gamma(\lambda)(2\alpha)^{\lambda - \frac{1}{2}}} |x - \mu|^{\lambda - \frac{1}{2}} K_{\lambda - \frac{1}{2}}(\alpha |x - \mu|) \exp(\beta |x - \mu|) \quad (21)$$

$$M_{VG}(t) = e^{\mu t} \left[\frac{\alpha^2 - \beta^2}{\alpha^2 - (\beta + t)^2} \right]^\lambda$$

For $\mu = \beta = 0$, $\alpha = 1/2$, and $\lambda = \nu/2$ the MGF reduces to Equation (20), the mean $\mu = 0$, and the PDF reduces to

$$\phi_{VCS}(u) = \frac{1}{2^\nu \sqrt{\pi} \Gamma(\nu/2)} |u|^{(\nu-1)/2} K_{(\nu-1)/2}\left(\frac{1}{2}|u|\right) \quad (22)$$

in which ν is degrees of freedom. We call this distribution the Variance Chi-Square, since it is a special case of the Variance-Gamma distribution^[10]. Here Γ is the Gamma function and K_λ is a modified Bessel function of the second kind. A Variance Chi-Square random variable is symmetric about $u = 0$, indicating that a background noise sample difference is positive or negative with equal likelihood.

A.5 Approximate normal distribution of the background noise sample difference

For a large number of independent frequency components, the background noise sample difference follows a normal distribution. To see this, evaluate the difference variable u of Equation (19) using Equation (17):

$$u = w_2 - w_1 = \sum_{k=1}^K y_k - \sum_{l=1}^L y_l \quad (23)$$

The expression may be rewritten as:

$$\frac{u}{\sqrt{M}\sigma_y} = \left(\frac{\sum_k y_k - \mu_y}{\sqrt{M}\sigma_y} \right) - \left(\frac{\sum_l y_l - \mu_l}{\sqrt{M}\sigma_l} \right) \quad (24)$$

Each of the two parenthetical terms follows a normal distribution with zero mean and unit variance, by the Central Limit Theorem^[6]. The standard deviation $\sigma_y = 2$ because y is Chi-Square with two degrees of freedom. The difference of the two terms follows a normal distribution with zero mean and variance two, by the summation property of the normal distribution. The normalized variable $z \equiv u / (\sqrt{2M}\sigma_y) = u / \sqrt{8M}$ therefore follows a zero mean, unit variance Normal distribution. The normal approximate PDF for the background noise sample difference is therefore

$$\phi(u) \approx \frac{1}{\sqrt{8M}} \phi_N\left(\frac{u}{\sqrt{8M}}\right) \quad (25)$$

in which $\phi_N(z)$ is the standard normal distribution. Figure 1 plots the exact VCS distribution, Equation (21), and its normal approximation and shows the approximation to be quite accurate for $M \geq 10$ frequency components.

A.6 Statistical background noise correction

To form the statistical background noise correction, Equations (10) and (19) are arranged to express the source estimate

$$\hat{s}^2 = s^2 + \frac{u_\alpha}{2M} \mu_{p^2} \quad (26)$$

in which the percentile u_α at confidence value α is associated with the background noise sample difference distribution by Equation (5). Noting $s^2 = m^2 - p_1^2$ by Equation (10), normalizing by \hat{s}^2 , and approximating the background noise sample p_1^2 and the background noise mean μ_{p^2} by the background noise mean estimate $\hat{\mu}_{p^2}$ gives

$$\frac{\hat{s}^2}{m^2} = 1 - \frac{\hat{\mu}_{p^2}}{m^2} \left(1 - \frac{u_\alpha}{2M} \right) \quad (27)$$

In terms of levels, the relationship is

$$L_p = L'_p + 10 \lg \left[1 - 10^{-\Delta/10} \left(1 - \frac{u_\alpha}{2M} \right) \right] \quad (28)$$

in which L_p is the source level estimate and L'_p is measured source level, and $u_\alpha = \Phi^{-1}(\alpha)$, where Φ^{-1} is the inverse cumulative probability distribution. Comparing Equations (1) and (28) reveals the statistical background noise correction, Equation (4).



Annex B (normative)

Case Studies and Examples

Alternate and standard background noise corrections are compared for background noise sampled in actual manufacturer and hypothetical example chambers. The identity of the manufacturer chambers is not revealed at the request of certain manufacturers. Successively measured background noise samples were selected to represent background noise sampling that might occur around source measurements. The background noise data should not be construed to represent future background noise conditions, because background noise may change over time. Background noise data for two example chambers, A and B, were synthesized to contrast with the lower and more repeatable background noise data of the actual anechoic chambers.

Background noise statistics are given in Table 1. Background noise steadiness M increases with background noise repeatability. Manufacturer A has the lowest and least steady background noise of the three chambers; however, some of the variation originates from rounding of background noise level to the nearest 0,1 dB. Manufacturer C has the highest and most steady background noise.

Table 1 also shows two sets of background noise samples for hypothetical chambers, labelled “Example A” and “Example B”. The M values for these hypothetical chambers are more than an order of magnitude smaller than those of the manufacturer chambers.

Background noise corrections K_1 and K_{ALT} for the 95% confidence value are plotted for the actual and example chambers in Figure 3. The values of the alternate correction have not been limited by the application of limits in Equation (3). The alternate correction K_{ALT} depends not only on the measured source-background noise level difference Δ , like the standard correction K_1 , but also on background noise steadiness M ; see Equations (6)-(8). Curves for K_{ALT} are shown for the Manufacturer A and Manufacturer C. The K_{ALT} values for Manufacturer B are omitted because they are nearly identical to those of Manufacturer C. The figure also shows the K_1 correction with caps K_{1max} of 0,46 dB and 1,26 dB, as prescribed by various standards^[1,2,3,4].

The behaviour of K_{ALT} relative to K_1 highlights the assumptions underlying each background noise correction. As background noise steadiness increases, the condition of zero background noise variation is approached. In Figure 3, K_{ALT} approaches K_1 as and background noise steadiness M increases towards the condition of completely steady background noise with zero variation that is tacitly built into the standard correction K_1 .

In reality background noise is unsteady, varying over time, and measured background noise may differ from the background noise contribution in a measured source level. Through its description of background noise variation, the background noise correction K_{ALT} provides several benefits over the standard correction K_1 :

- K_{ALT} minimizes risk of understating true source level by providing a background noise corrected source level (BNCSL) that upper bounds the true source level with 95% confidence, providing more assurance than the 50% confidence of the standard correction, when it is uncapped.
- K_{ALT} provides a single background noise correction across all chamber precision grades, unlike the chamber grade dependent standard background noise correction^[1,2,3,4], which results in a chamber precision penalty.
- K_{ALT} produces lower BNCSL values than K_1 for steady background noise within 6 to 10 dB of measured source level. In this case the alternate background noise correction is roughly 1-2 dB larger than the capped standard background noise correction, depending on the value of the cap K_{1max} .

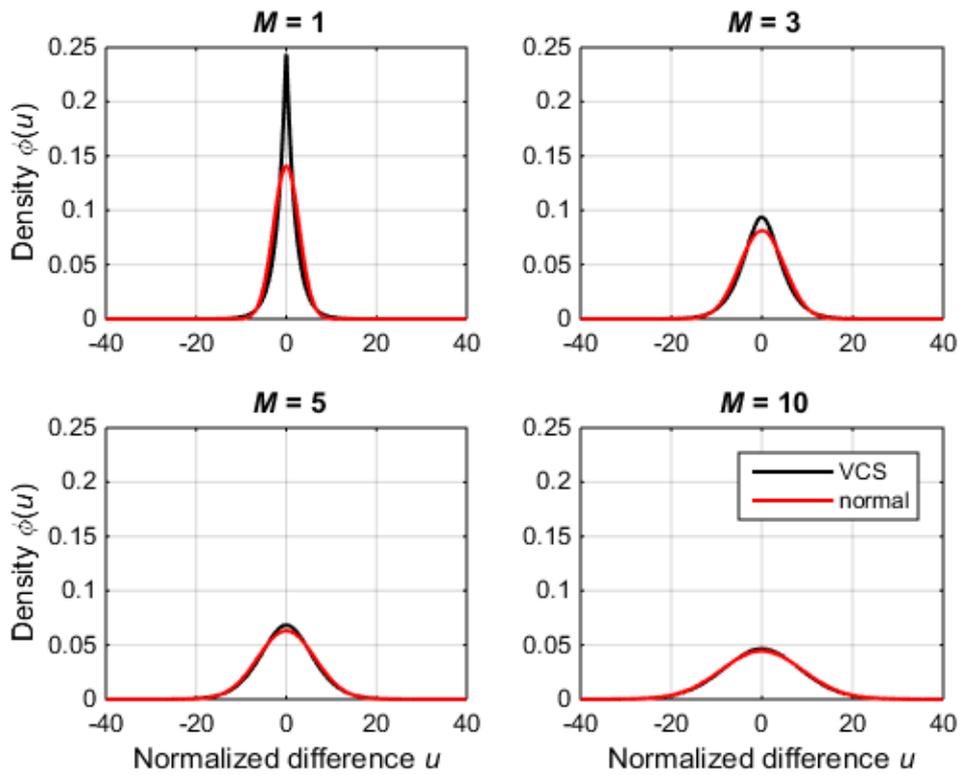


Figure B.1 — VCS and approximate normal probability density distributions

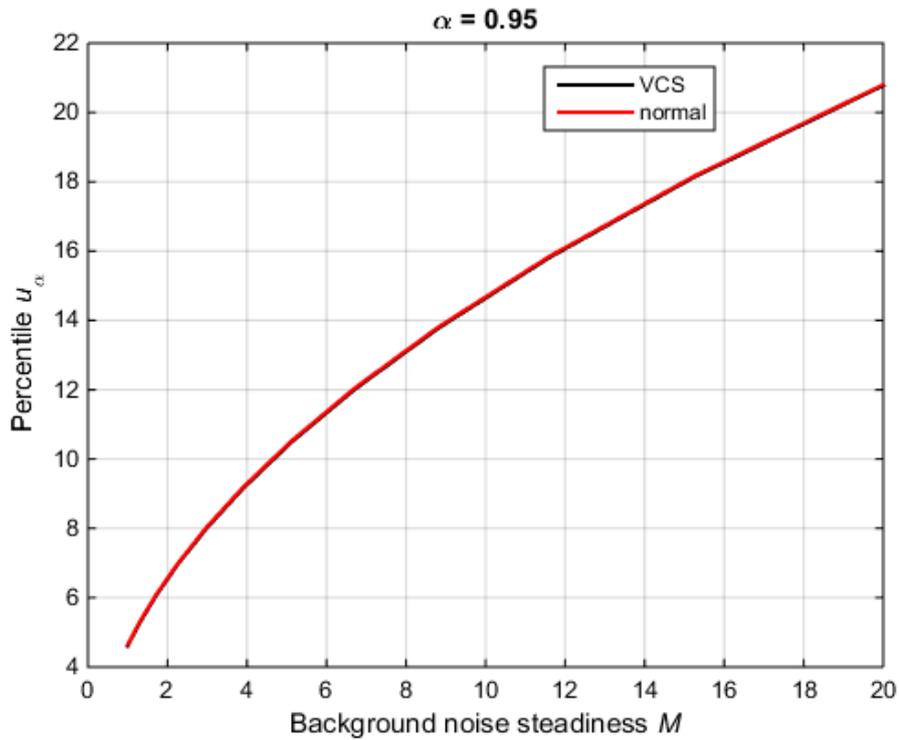


Figure B.2 — VCS and approximate normal percentiles, 95% confidence

Table B.1 — Background noise steadiness by chamber

Chamber	Background Noise Level (dB)				Background Noise (20 μ Pa) ²		Background Noise Steadiness M
	Sample 1	Sample 2	Sample 3	Sample 4	Mean	Standard Deviation	
Manufacturer A	6,9	7,0	7,5	7,3	5,2	0,33	250
Manufacturer B	16,56	16,75	16,56	16,57	45,8	0,975	2210
Manufacturer C	21,74	21,78	21,76	21,91	151	2,61	3360
Example A	30,0	30,8	31,6	32,4	1300	320	18
Example B	40,0	41,5	43,0	44,5	18000	7900	5,2

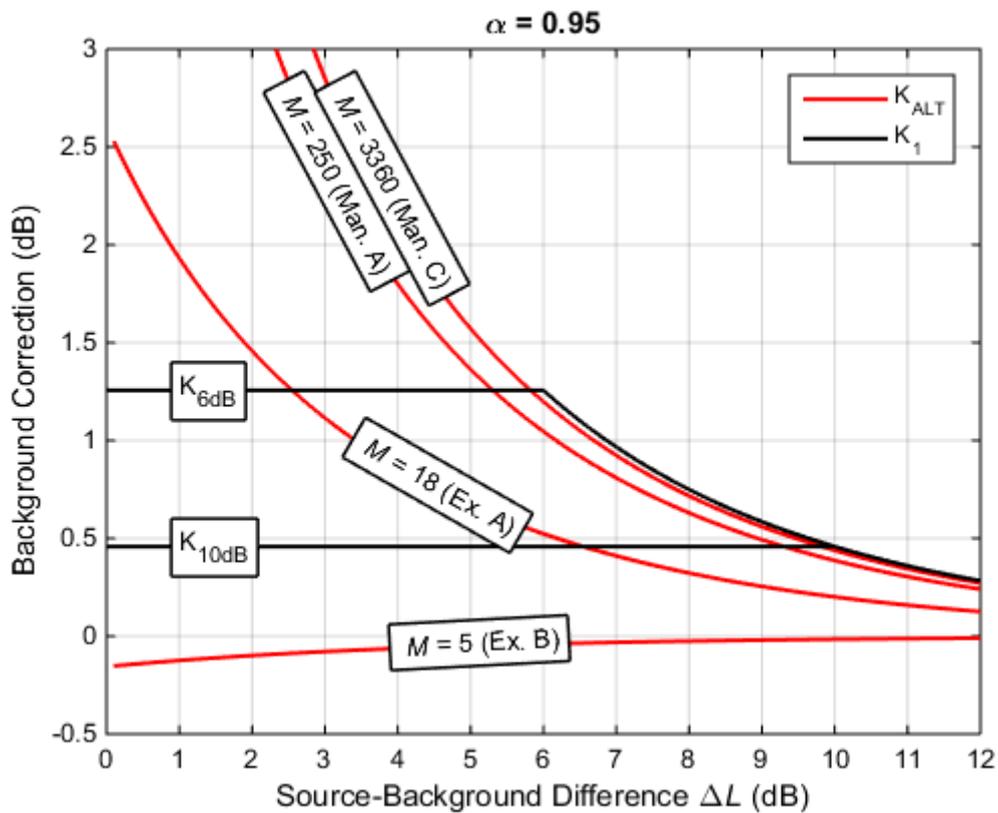


Figure B.3 — Alternate K_{ALT} and standard K_1 background noise corrections



Annex C (normative)

Calculation of Background Noise Steadiness

The background noise steadiness M is calculated from samples of background noise. Background noise is the average over time of squared pressure and is expressed in squared Pascals. Background noise is different than background noise level, which is expressed in dB.

The steps for obtaining the steadiness M are:

- Determine the background noise mean $\hat{\mu}_{p^2}$ by Equation (7)
- Determine the background noise standard deviation $\hat{\sigma}_{p^2}^2$ by Equation (8)
- Determine the steadiness M by Equation (6)

These steps use background noise, not background noise level. Background noise is related to background noise level by $p_i^2 = p_0^2 10^{L_i/10}$ in which p_i^2 is background noise, L_i is background noise level, and p_0 is the level reference. Although the level reference is standardized as 20 μPa , the convenient value of unity may be used instead because the level reference cancels in the ratio of Equation (6).

An example of determining background noise steadiness is given in Table C.1, using the data from Table 1.

Table C.1 — Background noise by chamber

Chamber	Background Noise Level (dB)				Background Noise (20 μPa) ²						Steadiness
	S1	S2	S3	S4	S1	S2	S3	S4	Mean	Std. dev.	
Manufacturer A	6.9	7.0	7.5	7.3	4.9	5.0	5.6	5.4	5,2	0,33	250
Manufacturer B	16.56	16.75	16.56	16.57	45	47	45	45	45,8	0,975	2210
Manufacturer C	21.74	21.78	21.76	21.91	149	151	150	155	151	2,61	3360
Example A	30,0	30,8	31,6	32,4	1000	1202	1445	1738	1300	320	18
Example B	40,0	41,5	43,0	44,5	10000	14125	19953	28184	18000	7900	5,2



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